Practice Test 3

AP® Calculus AB Exam

SECTION I: Multiple-Choice Questions

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 45 minutes
Number of Questions
45
Percent of Total Grade
50%
Writing Instrument
Pencil required

Instructions

Section I of this examination contains 45 multiple-choice questions. Fill in only the ovals for numbers 1 through 45 on your answer sheet.

CALCULATORS MAY NOT BE USED IN THIS PART OF THE EXAMINATION.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent	Ⓐ ● Ⓒ Ⓓ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all the multiple-choice questions.

About Guessing

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. Multiple choice scores are based on the number of questions answered correctly. Points are not deducted for incorrect answers, and no points are awarded for unanswered questions. Because points are not deducted for incorrect answers, you are encouraged to answer all multiple-choice questions. On any questions you do not know the answer to, you should eliminate as many choices as you can, and then select the best answer among the remaining choices.

CALCULUS AB

SECTION I, Part A

Time—60 Minutes

Number of questions—30

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_{\frac{\pi}{4}}^{x} \cos(2t)\,dt =$

(A) $\cos(2x)$

(B) $\dfrac{\sin(2x) - 1}{2}$

(C) $\cos(2x) - 1$

(D) $\dfrac{\sin 2(x)}{2}$

---

2. What are the coordinates of the point of inflection on the graph of $y = x^3 - 15x^2 + 33x + 100$?

(A) (9, 0)
(B) (5, –48)
(C) (9, –89)
(D) (5, 15)

GO ON TO THE NEXT PAGE.

3. If $3x^2 - 2xy + 3y = 1$, then when $x = 2$, $\frac{dy}{dx} =$

(A) -12

(B) -10

(C) $-\frac{10}{7}$

(D) 12

---

4. $\int_1^3 \frac{8}{x^3}\,dx =$

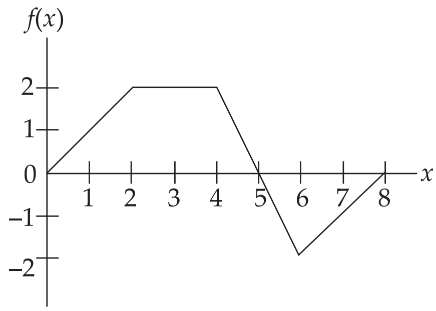
(A) $\frac{32}{9}$

(B) $\frac{40}{9}$

(C) 0

(D) $-\frac{32}{9}$

GO ON TO THE NEXT PAGE.

5. 

The graph of a piecewise linear function f, for $0 \leq x \leq 8$, is shown above. What is the value of $\int_0^8 f(x)\,dx$?

(A) 1
(B) 4
(C) 8
(D) 10

---

6. $\lim_{x \to 0} \frac{x - \sin x}{x^3} =$

(A) 0
(B) 1
(C) 2
(D) Does not exist

GO ON TO THE NEXT PAGE.

7. If $f(x) = x^2\sqrt{3x+1}$, then $f'(x) =$

(A) $\dfrac{9x^2+2x}{\sqrt{3x+1}}$

(B) $\dfrac{-9x^2+4x}{2\sqrt{3x+1}}$

(C) $\dfrac{15x^2+4x}{2\sqrt{3x+1}}$

(D) $\dfrac{-9x^2-4x}{2\sqrt{3x+1}}$

---

8. What is the instantaneous rate of change at $t = -1$ of the function f, if $f(t) = \dfrac{t^3+t}{4t+1}$?

(A) $\dfrac{12}{9}$

(B) $\dfrac{4}{9}$

(C) $-\dfrac{4}{9}$

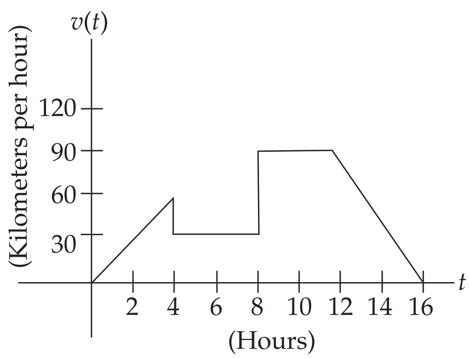
(D) $-\dfrac{12}{9}$

GO ON TO THE NEXT PAGE.

9. $\int_{2}^{e+1}\left(\frac{4}{x-1}\right)dx =$

(A) 4
(B) $4e$
(C) 0
(D) –4

10.



A car's velocity is shown on the graph above. Which of the following gives the total distance traveled from $t = 0$ to $t = 16$ (in kilometers)?

(A) 360
(B) 390
(C) 780
(D) 1,000

GO ON TO THE NEXT PAGE.

11. $\dfrac{d}{dx}\tan^2(4x) =$

(A) $8\tan(4x)$
(B) $4\sec^4(4x)$
(C) $8\tan(4x)\sec^2(4x)$
(D) $4\tan(4x)\sec^2(4x)$

---

12. What is the equation of the line tangent to the graph of $y = \sin^2 x$ at $x = \dfrac{\pi}{4}$?

(A) $y - \dfrac{1}{2} = \left(x - \dfrac{\pi}{4}\right)$

(B) $y - \dfrac{1}{\sqrt{2}} = \left(x - \dfrac{\pi}{4}\right)$

(C) $y - \dfrac{1}{\sqrt{2}} = \dfrac{1}{2}\left(x - \dfrac{\pi}{4}\right)$

(D) $y - \dfrac{1}{2} = \dfrac{1}{2}\left(x - \dfrac{\pi}{4}\right)$

GO ON TO THE NEXT PAGE.

13. If the function $f(x) = \begin{cases} 3ax^2 + 2bx + 1; & x \le 1 \\ ax^4 - 4bx^2 - 3x; & x > 1 \end{cases}$ is differentiable for all real values of x, then $b =$

(A) $-\frac{11}{4}$

(B) $\frac{1}{4}$

(C) 0

(D) $-\frac{1}{4}$

---

14. The graph of $y = x^4 + 8x^3 - 72x^2 + 4$ is concave down for

(A) $-6 < x < 2$

(B) $x > 2$

(C) $x < -6$

(D) $-3 - 3\sqrt{5} < x < -3 + 3\sqrt{5}$

---

15. $\lim\limits_{x\to\infty} \frac{\ln(x+1)}{\log_2 x} =$

(A) $\frac{1}{\ln 2}$

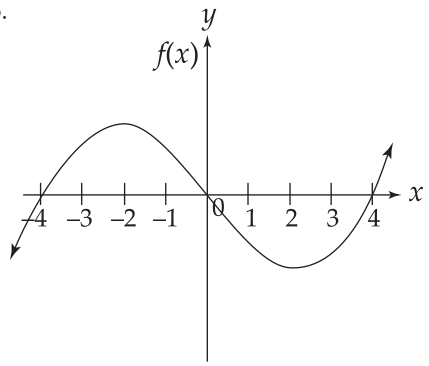
(B) 0

(C) 1

(D) $\ln 2$

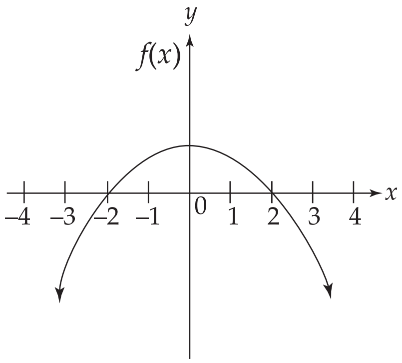
GO ON TO THE NEXT PAGE.

16.

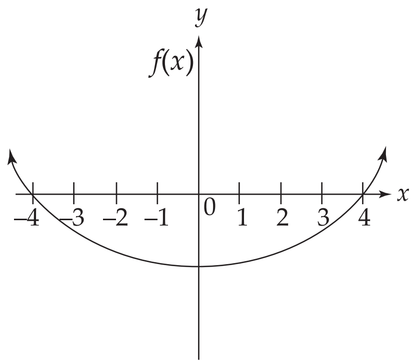


The graph of $f(x)$ is shown in the figure above. Which of the following could be the graph of $f'(x)$?

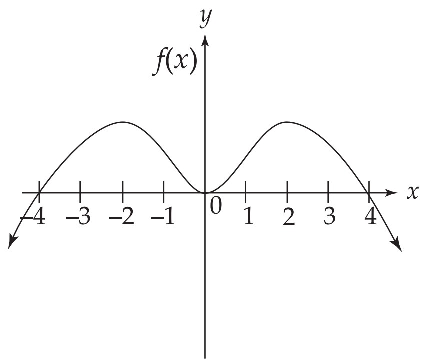
(A)



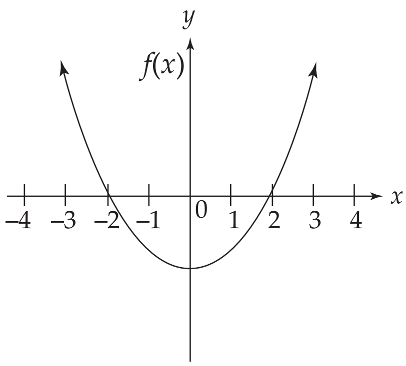
(C)



(B)



(D)



GO ON TO THE NEXT PAGE.

17. If $f(x) = \ln(\cos(3x))$, then $f'(x) =$

(A) $3\sec(3x)$
(B) $3\tan(3x)$
(C) $-3\tan(3x)$
(D) $-3\cot(3x)$

---

18. If $f(x) = \int_0^{x+1} \sqrt[3]{t^2 - 1}\, dt$, then $f'(-4) =$

(A) -2

(B) 2

(C) $\sqrt[3]{15}$

(D) 0

---

19. A particle moves along the x-axis so that its position at time t, in seconds, is given by $x(t) = t^2 - 7t + 6$. For what value(s) of t is the velocity of the particle zero?

(A) 1
(B) 6
(C) 1 or 6
(D) 3.5

GO ON TO THE NEXT PAGE.

20. $\int_0^{\frac{\pi}{2}} \sin(2x)e^{\sin^2 x}\,dx =$

(A) $e - 1$
(B) $1 - e$
(C) $e + 1$
(D) 1

---

21. The average value of $\sec^2 x$ on the interval $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$ is

(A) $\frac{12\sqrt{3} - 12}{\pi}$

(B) $\frac{12 - 4\sqrt{3}}{\pi}$

(C) $\frac{6\sqrt{2} - 6}{\pi}$

(D) $\frac{6 - 6\sqrt{2}}{\pi}$

---

22. Find the area of the region bounded by the parabolas $y = x^2$ and $y = 6x - x^2$.

(A) 9
(B) 27
(C) –9
(D) –18

GO ON TO THE NEXT PAGE.

23. The function f is given by $f(x) = x^4 + 4x^3$. On which of the following intervals is f decreasing?

(A) $(-3, 0)$

(B) $(0, \infty)$

(C) $(-3, \infty)$

(D) $(-\infty, -3)$

---

24. $\lim_{x \to 0} \dfrac{\tan(3x) + 3x}{\sin(5x)} =$

(A) 0

(B) $\dfrac{3}{5}$

(C) $\dfrac{6}{5}$

(D) Nonexistent

---

25. If the region enclosed by the y-axis, the curve $y = 4\sqrt{x}$, and the line $y = 8$ is revolved about the x-axis, the volume of the solid generated is

(A) $\dfrac{32\pi}{3}$

(B) 128π

(C) $\dfrac{128}{3}$

(D) $\dfrac{128\pi}{3}$

GO ON TO THE NEXT PAGE.

26. The maximum velocity attained on the interval $0 \leq t \leq 5$, by the particle whose displacement is given by $s(t) = 2t^3 - 12t^2 + 16t + 2$ is

(A) 286
(B) 46
(C) 16
(D) 0

---

27. The value of c that satisfies the Mean Value Theorem for derivatives on the interval [0, 5] for the function $f(x) = x^3 - 6x$ is

(A) 0

(B) 1

(C) $\frac{5}{3}$

(D) $\frac{5}{\sqrt{3}}$

---

28. If $f(x) = \sec(4x)$, then $f'\left(\frac{\pi}{16}\right)$ is

(A) $4\sqrt{2}$

(B) $\sqrt{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{4}{\sqrt{2}}$

GO ON TO THE NEXT PAGE.

29. $\frac{d}{dx}\int_{2x}^{5x} \cos t \, dt =$

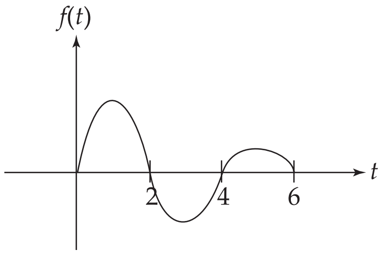
(A) $5 \cos 5x - 2 \cos 2x$

(B) $5 \sin 5x - 2 \sin 2x$

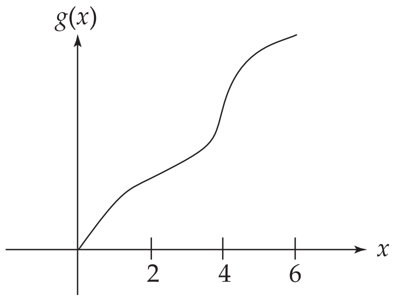
(C) $\cos 5x - \cos 2x$

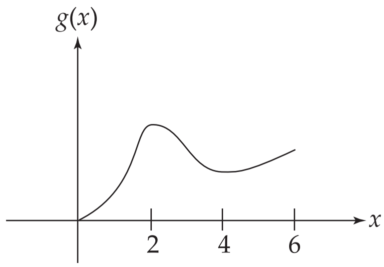
(D) $\sin 5x - \sin 2x$

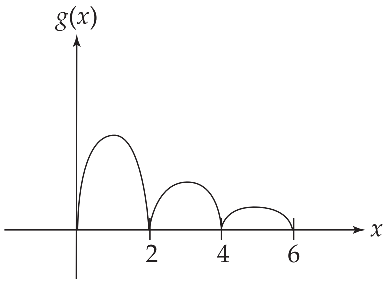
GO ON TO THE NEXT PAGE.

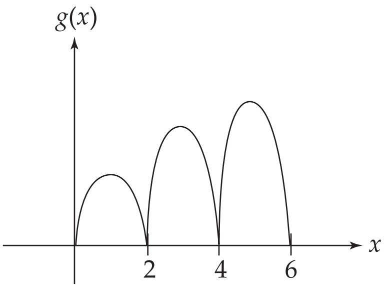
30. 

Let $g(x) = \int_0^x f(t)\, dt$, where $f(t)$ has the graph shown above. Which of the following could be the graph of g ?

(A) 

(B) 

(C) 

(D) 

END OF PART A, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION I, Part B

Time—45 Minutes

Number of questions—15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

31. If $f(x)$ is the function given by $f(x) = e^{3x} + 1$, at what value of x is the slope of the tangent line to $f(x)$ equal to 2 ?

(A) –0.135
(B) 0
(C) –0.366
(D) 0.693

GO ON TO THE NEXT PAGE.

32. The graph of the function $y = x^3 + 12x^2 + 15x + 3$ has a relative maximum at $x =$

(A) –10.613
(B) –0.248
(C) –7.317
(D) –1.138

---

33. The side of a square is increasing at a constant rate of 0.4 cm/sec. In terms of the perimeter, P, what is the rate of change of the area of the square, in cm^2/sec?

(A) $0.05P$
(B) $0.2P$
(C) $0.4P$
(D) $51.2P$

---

34. Let f be the function given by $f(x) = 3^x$. For what value of x is the slope of the line tangent to the curve at $(x, f(x))$ equal to 1 ?

(A) 0.086
(B) 0
(C) –0.086
(D) –1.099

GO ON TO THE NEXT PAGE.

35. Given f and g are differentiable functions and

$$f(a) = -4,\ g(a) = c,\ g(c) = 10,\ f(c) = 15$$

$$f'(a) = 8,\ g'(a) = b,\ g'(c) = 5,\ f'(c) = 6$$

If $h(x) = f(g(x))$, find $h'(a)$

(A) $6b$
(B) $8b$
(C) 80
(D) $15b$

---

36. What is the area of the region in the first quadrant enclosed by the graph of $y = e^{-\frac{x^2}{4}}$ and the line $y = 0.5$?

(A) 0.240
(B) 0.516
(C) 0.480
(D) 1.032

---

37. What is the trapezoidal approximation of $\int_0^3 e^x\,dx$ using $n = 4$ subintervals?

(A) 6.407
(B) 13.565
(C) 19.972
(D) 27.879

GO ON TO THE NEXT PAGE.

38. The second derivative of a function f is given by $f''(x) = x \sin x - 2$. How many points of inflection does f have on the interval $(-10, 10)$?

(A) Zero
(B) Two
(C) Four
(D) Eight

---

39. $\lim_{h \to 0} \dfrac{\sin\left(\dfrac{5\pi}{6} + h\right) - \dfrac{1}{2}}{h} =$

(A) $\dfrac{\sqrt{3}}{2}$

(B) $\dfrac{1}{2}$

(C) $-\dfrac{1}{2}$

(D) $-\dfrac{\sqrt{3}}{2}$

---

40. The base of a solid S is the region enclosed by the graph of $4x + 5y = 20$, the x-axis, and the y-axis. If the cross-sections of S perpendicular to the x-axis are semicircles, then the volume of S is

(A) $\dfrac{5\pi}{3}$

(B) $\dfrac{10\pi}{3}$

(C) $\dfrac{50\pi}{3}$

(D) $\dfrac{225\pi}{3}$

GO ON TO THE NEXT PAGE.

41. Which of the following is an equation of the line tangent to the graph of $y = x^3 + x^2$ at $y = 3$?

(A) $y = 33x - 63$
(B) $y = 33x - 13$
(C) $y = 6.488x - 4.620$
(D) $y = 6.488x - 10.620$

---

42. If $f'(x) = \ln x - x + 2$, at which of the following values of x does f have a relative minimum value?

(A) 5.146
(B) 3.146
(C) 0.159
(D) 0

---

43. Find the total area of the region between the curve $y = \cos x$ and the x-axis from $x = 1$ to $x = 2$ in radians.

(A) 0
(B) 0.068
(C) 0.249
(D) 1.751

GO ON TO THE NEXT PAGE.

44. Let $f(x) = \int \cot x \, dx$; $0 < x < \pi$. If $f\left(\frac{\pi}{6}\right) = 1$, then $f(1) =$

(A) –1.861
(B) –0.480
(C) 0.524
(D) 1.521

---

45. A radioactive isotope, y, decays according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in seconds. If the half-life of y is 1 minute, then the value of k is

(A) –41.589
(B) –0.012
(C) 0.027
(D) 0.693

STOP

END OF PART B, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

SECTION II
GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt (X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

SECTION II, PART A
Time—30 minutes
Number of problems—2

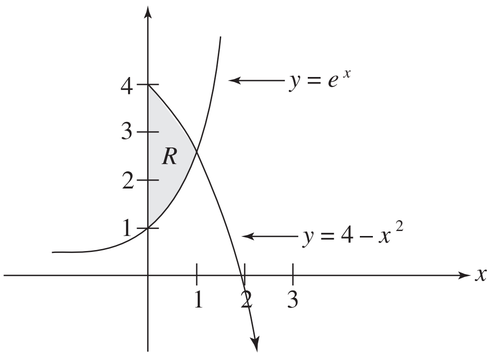
A graphing calculator is required for some problems or parts of problems.

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

GO ON TO THE NEXT PAGE.

1.



Let R be the region in the first quadrant shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) Find the volume of the solid generated when R is revolved about the line $x = -1$.

GO ON TO THE NEXT PAGE.

2. A body is coasting to a stop and the only force acting on it is a resistance proportional to its speed, according to the equation $\frac{ds}{dt} = v_f = v_0 e^{-\left(\frac{k}{m}\right)t}$; $s(0) = 0$, where v_0 is the body's initial velocity (in m/s), v_f is its final velocity, m is its mass, k is a constant, and t is time.

(a) If a body with mass $m = 50$ kg and $k = 1.5$ kg/sec initially has a velocity of 30 m/s, how long, to the nearest second, will it take to slow to 1 m/s?

(b) How far, to the 10 nearest meters, will the body coast during the time it takes to slow from 30 m/s to 1 m/s?

(c) If the body coasts from 30 m/s to a stop, how far will it coast?

GO ON TO THE NEXT PAGE.

SECTION II, PART B
Time—1 hour
Number of problems—4

No calculator is allowed for these problems.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

---

3. An object moves with velocity $v(t) = 9t^2 + 18t - 7$ for $t \geq 0$ from an initial position of $s(0) = 3$.

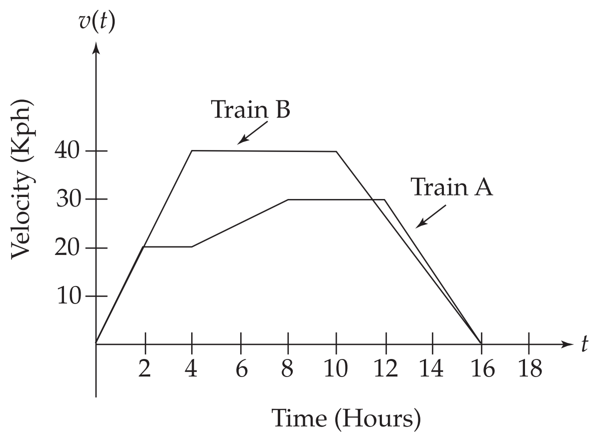
   (a) Write an equation for the position of the particle.

   (b) When is the particle changing direction?

   (c) What is the total distance covered from $t = 2$ to $t = 5$?

GO ON TO THE NEXT PAGE.

4.



Three trains, *A, B,* and *C* each travel on a straight track for $0 \leq t \leq 16$ hours. The graphs above, which consist of line segments, show the velocities, in kilometers per hour, of trains *A* and *B*. The velocity of *C* is given by

$$v(t) = 8t - 0.25t^2$$

(Indicate units of measure for all answers.)

(a) Find the velocities of *A* and *C* at time $t = 6$ hours.

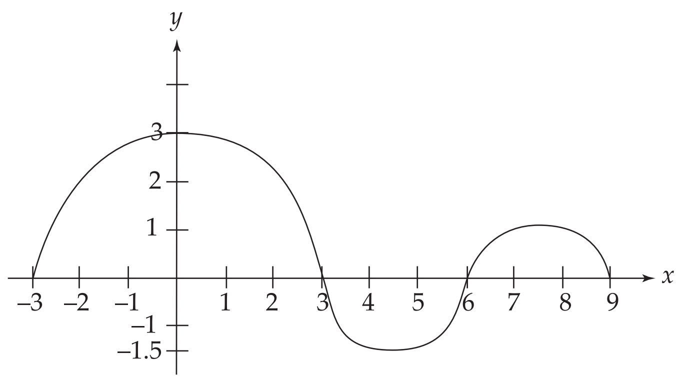
(b) Find the accelerations of *B* and *C* at time $t = 6$ hours.

(c) Find the positive difference between the total distance that *A* traveled and the total distance that *B* traveled in 16 hours.

(d) Find the total distance that *C* traveled in 16 hours.

GO ON TO THE NEXT PAGE.

5.



The figure above shows the graph of $g(x)$, where g is the derivative of the function f, for $-3 \leq x \leq 9$. The graph consists of three semicircular regions and has horizontal tangent lines at $x = 0$, $x = 4.5$, and $x = 7.5$.

(a) Find all values of x, for $-3 < x \leq 9$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-3 < x \leq 9$, at which f attains a relative maximum. Justify your answer.

(c) If $f(x) = \int_{-3}^{x} g(t)\, dt$, find $f(6)$.

(d) Find all points where $f''(x) = 0$.

GO ON TO THE NEXT PAGE.

6. Consider the curve given by $x^2y - 4x + y^2 = 2$.

(a) Find $\dfrac{dy}{dx}$.

(b) Find $\dfrac{d^2y}{dx^2}$.

(c) Find the equation of the tangent lines at each of the two points on the curve whose x-coordinate is 1.

STOP

END OF EXAM

The Princeton Review®

Completely darken bubbles with a No. 2 pencil. If you make a mistake, be sure to erase mark completely. Erase all stray marks.

1.

YOUR NAME: ____
(Print) Last First M.I.

SIGNATURE: ______________________________ DATE: ___/___/___

HOME ADDRESS: ____
(Print) Number and Street

____
City State Zip Code

PHONE NO.: ____

IMPORTANT: Please fill in these boxes exactly as shown on the back cover of your test book.

2. TEST FORM

____

3. TEST CODE

0	A	J	0	0
1	B	K	1	1
2	C	L	2	2
3	D	M	3	3
4	E	N	4	4
5	F	O	5	5
6	G	P	6	6
7	H	Q	7	7
8	I	R	8	8
9			9	9

4. REGISTRATION NUMBER

0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

5. YOUR NAME

First 4 letters of last name					FIRST INIT	MID INIT
A	A	A	A		A	A
B	B	B	B		B	B
C	C	C	C		C	C
D	D	D	D		D	D
E	E	E	E		E	E
F	F	F	F		F	F
G	G	G	G		G	G
H	H	H	H		H	H
I	I	I	I		I	I
J	J	J	J		J	J
K	K	K	K		K	K
L	L	L	L		L	L
M	M	M	M		M	M
N	N	N	N		N	N
O	O	O	O		O	O
P	P	P	P		P	P
Q	Q	Q	Q		Q	Q
R	R	R	R		R	R
S	S	S	S		S	S
T	T	T	T		T	T
U	U	U	U		U	U
V	V	V	V		V	V
W	W	W	W		W	W
X	X	X	X		X	X
Y	Y	Y	Y		Y	Y
Z	Z	Z	Z		Z	Z

6. DATE OF BIRTH

Month	Day		Year	
JAN				
FEB	0	0	0	0
MAR	1	1	1	1
APR	2	2	2	2
MAY	3	3	3	3
JUN		4	4	4
JUL		5	5	5
AUG		6	6	6
SEP		7	7	7
OCT		8	8	8
NOV		9	9	9
DEC				

7. GENDER

MALE

FEMALE

The Princeton Review®

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E
16. A B C D E
17. A B C D E
18. A B C D E
19. A B C D E
20. A B C D E
21. A B C D E
22. A B C D E
23. A B C D E
24. A B C D E
25. A B C D E
26. A B C D E
27. A B C D E
28. A B C D E
29. A B C D E
30. A B C D E
31. A B C D E
32. A B C D E
33. A B C D E
34. A B C D E
35. A B C D E
36. A B C D E
37. A B C D E
38. A B C D E
39. A B C D E
40. A B C D E
41. A B C D E
42. A B C D E
43. A B C D E
44. A B C D E
45. A B C D E